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MATHEMATICAL MODELING OF TAMBAQUI GROWTH PATTERN THROUGH THE GOMPERTZ MODEL

MODELAGEM MATEMÁTICA DO PADRÃO DE CRESCIMENTO DO TAMBAQUI POR MEIO DO MODELO DE GOMPERTZ

Laina Pires ROSA[1]; Anne Caroline Mascarenhas dos SANTOS[1]; Larissa de Souza BARROS[1]; Tania Sakai Rodrigues PEIXOTO[1]; Kerolayne Cristina GONÇALVES[1]; Leandra Cristina Crema CRUZ[1] & Pedro Alexandre da CRUZ[1]*

[1]Universidade Federal do Tocantins - UFT

*email: pedrocruz@mail.uft.edu.br

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Resumo - O padrão de crescimento do tambaqui (*Colossoma macropomum*) foi avaliado pelo modelo matemático de Gompertz. Diferentes curvas foram construídas para conferir peso corporal, comprimento da cabeça, altura do corpo e comprimento total do peixe, sendo tais parâmetros analisados durante oito meses. Projeções das quatro características morfológicas de *C. macropomum* foram realizadas da fase alevino/juvenil até a engorda. A modelagem teve por finalidade fornecer uma estimativa do padrão de crescimento de uma população de tambaqui cultivada em tanque-rede no estado do Tocantins, Brasil. Conforme o modelo de Gompertz, o padrão de crescimento corporal do tambaqui foi estimado e a construção das projeções indicou que o peixe continua em fase de crescimento após atingir as características consideradas para abate. Além disso, o modelo estimou o comprimento corporal máximo do tambaqui (87,61 cm) avaliando também que a espécie levaria aproximadamente 20 anos para atingir esse tamanho. O objetivo deste trabalho foi estudar o padrão de crescimento corporal de *C. macropomum* e contribuir para o desenvolvimento de novos métodos de cultivo que visem à otimização da produção de tal peixe.

Palavras-Chave: aquicultura, *Colossoma macropomum*, modelagem estatística.

Abstract - Tambaqui (*Colossoma macropomum*) growth behavior was evaluated through the Gompertz mathematical model. Different curves were built for the body weight, head length, body height and body length, which were measured for eight months. Projections of the four morphological characteristics of *C. macropomum* from the fry/juvenile until fattening stage. The modeling aimed to estimate the growth pattern of the tambaqui population being cultivated in net-tanks in the state of Tocantins, Brazil. Through the Gompertz model, the corporal growth pattern of tambaqui was estimated and the construction of projection curves indicated that the fish is still in growth phase after reaching the features being considered to slaughter. Also, the model predicted the maximum corporal length of tambaqui (87.61 cm) and that the fish would take approximately 20 years to achieve it. The goal of the present work was to study the bodily growth pattern of *C. macropomum* and contribute to the development of novel cultivation methods to optimize the production of such fish.

Keywords: aquaculture, *Colossoma macropomum*, statistic modelling.

Introduction

Colossoma macropomum, also known as tambaqui, is a freshwater fish native from the North of Brazil that has high-quality meat and excellent productivity; regarding growth, it can reach up to 90 cm in length and 30 Kg of corporal weight (De Mello et al., 2015; Marcos, 2014; Mourad et al., 2018). Because of that, tambaqui is not only one of the main food sources in such region but also of interest for aquaculture. Brazil shows optimal natural conditions for the development of fish-farming, such as 5,500 ha of freshwater reservoir and a growing consumer market (Sebrae, n.d.). In 2012, IBAMA (Instituto Brasileiro do Meio Ambiente) authorized tambaqui farming at the hydrographic region of Tocantins-Araguaia, in artificial water reservoirs under proper licensing, and the Fishing Ministry implemented the Aquiculture Fishing Harvest Plan in the state of Tocantins (Conama, 2009). The latest IBGE (Instituto Brasileiro de Geografia e Estatística) aquaculture data showed that tambaqui was the second most produced fish in the country (approximately 137,000 tons), the North of Brazil alone was responsible for almost 80% of that production and the state of Tocantins held 4.5% (5,000 tons) of the tambaqui produced in the region (IBGE, 2016). Despite the large production of tambaqui, little is studied about its growth pattern and performance (Ariede et al., 2018).

With the large production of tambaqui in Brazil, the study of growth curve of such fish becomes necessary to evaluate its developmental behavior in different environments and optimize production (Mourad et al., 2018), as it has been done for tilapia (Ansah & Frimpong, 2015; De Lima Amancio, Da Silva, Fernandes, Sakomura & Da Cruz, 2014), the most produced fish in the country (IBGE, 2016). Mathematical modeling is commonly used to describe populational dynamics and have been largely used in aquaculture, such as Malthus, Verhulst and Gompertz (Cruz, Rosa & Cruz, 2016). In the Gompertz model, which is derived from Verhulst, the growth rate can be modified along the developmental phase of the fish (*e.g.*, from fry to adult) and considers that the corporal mass of the animal is always superior to zero, meaning that at the age zero the animal has weight and dimensions. Therefore, the adaptation of such model to describe the development of individuals within a population could be of great value to predict optimal harvest in fish-farms (Ansah & Frimpong, 2015). To better describe the growth pattern of *C. macropomum* and provide grounding for novel cultivation methods focused on the optimization of tambaqui production, we applied the adapted Gompertz model to different corporal features (weight, body length, body height and head length) of tambaqui produced in net-tanks in the state of Tocantins.

Materials and Methods

The experiment was performed at Granja Cariri (Cariri do Tocantins, Tocantins, Brazil), owned by Mr. Eduardo Sakai, from April to December of 2016. A total of 30 tambaqui in the fry stage with initial weight of approximately 5 g were cultivated in net-tanks and fed twice a day. Their body weight (P) and dimensions (Figure 1) were measured monthly, and the latter were classified as body length (CT), the distance between the anterior head extremity and the end of the caudal fin, head height (AC), starting from the 1st radius and ending on dorsal fin, and head length (TC), from the anterior head extremity to the caudal plate of operculum.

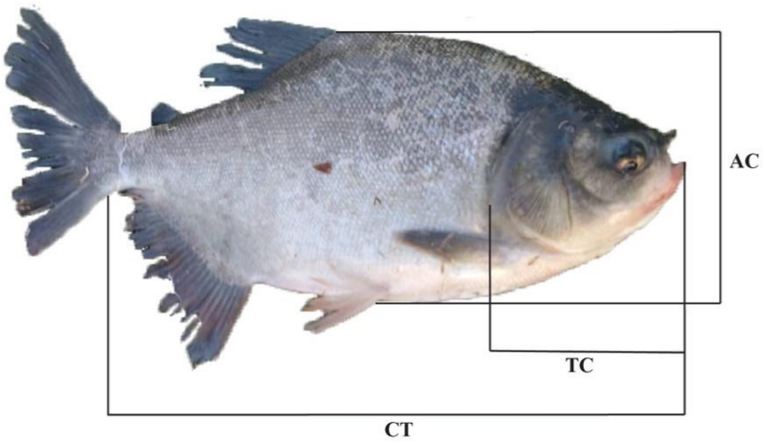

Figure 1. Parameters measured in tambaqui and considered for mathematical modeling with Gompertz: body length (CT), body height (AC) and head length (TC).

Gompertz model treats the variable's inhibition rate as proportional to the logarithm of that same variable. In other words, the growth rate is larger in the beginning of the process and decreases rapidly (Bassanezi, 2018). The Gompertz model is defined by the following differential Equation 1:

$$\frac{dP}{dt} = rNln\left(\frac{K}{N}\right) (1)$$

In which:
r = relative growth rate when K is small;
K = finite limit value –features for slaughter;
N = estimate weight (g) or size (cm).

The Gompertz model (Gompertz, 1825) was applied to stablish the growth pattern of tambaqui as described as Equation 2 (Filho, De Aguiar Neto & Da Silva, 2016):

$$N(t) = Ke^{(Ce^{-rt})} (2)$$

In which:
N(*t*) = estimate weigh (g) or size (cm) at the age *t*;
K = weigh or asymptotic size when *t* tends to the infinity;
C = relative growth at the inflexion point (cm/day per cm or g/day per g);
r = relative growth rate;
t = age (days).

Relative growth at the inflexion point (C) and relative growth rate (r) were respectively determined through the Equations 3 and 4:

$$C = ln\left(\frac{No}{K}\right) (3)$$

$$r = \frac{-1}{t} ln\left(\frac{1}{C} ln\left(\frac{N}{K}\right)\right) (4)$$

The variance of the experimental data was estimated through ANOVA (analysis of variance). The average of measurements for P, AC, CT and TC were compared individually by the Tukey's test with 5% confidence level, and are represented in Table 1 along with its respective standard deviation. Data processing and statistical analysis were performed through Minitab 16 (Minitab, 2015) and Excel (Microsoft, 2010).

Results and Discussion

The results of the statistical analysis for the experimental data obtained for body length (CT), head length (TC), body height (AC) and body weight (P) of the fish through the sampling period (age) are presented in Table 1.

Table 1. Average of the monthly measurements of the weight and morphological features of tambaqui.

Age (days)	Body length (cm)	Head length (cm)	Body height (cm)	Weight (g)
0	5.00 ± 0.23^{h}	0.50 ± 0.02^{g}	1.50 ± 0.04^{h}	5.00 ± 0.94^{i}
30	9.25 ± 0.86^{g}	1.04 ± 0.23^{f}	3.21 ± 0.55^{g}	30.00 ± 0.81^{h}
60	11.95 ± 0.55^{f}	1.57 ± 0.25^{e}	3.95 ± 0.47^{f}	42.00 ± 1.09^{g}
90	12.70 ± 1.08^{f}	1.67 ± 0.26^{e}	4.17 ± 0.61^{f}	49.95 ± 1.46^{f}
120	16.45 ± 1.48^{e}	2.52 ± 0.20^{d}	7.45 ± 0.49^{e}	125.96 ± 1.38^{e}
150	19.20 ± 0.90^{d}	3.01 ± 0.37^{c}	8.55 ± 0.41^{d}	151.20 ± 1.10^{d}
180	21.50 ± 0.36^{c}	4.60 ± 0.20^{b}	10.30 ± 0.33^{c}	240.50 ± 1.38^{c}
210	24.10 ± 0.42^{b}	5.90 ± 0.33^{a}	11.20 ± 0.26^{b}	299.00 ± 1.39^{b}
240	26.50 ± 0.42^{a}	6.20 ± 0.14^{a}	13.00 ± 0.49^{a}	340.00 ± 1.42^{a}
Coefficient of determination (R^2)	0.986	0.985	0.987	0.999
p-value	0.000	0.000	0.000	0.000

[abc]Body length, head length, body height or body weight of the fish followed by the same letter, in the vertical orientation, do not statistically differ based on the Tukey's test with 5% confidence level.

A progressive increase of all the measurements (CT, TC, AC and P) over age (0–240 days) was observed. During the first two months (60 and 90 days) the results for CT were not statistically different. For TC, the data did not differ for 60 and 90 days, nor 210 and 240 days. During the first months (60 and 90 days), AC measurements did not show divergence. All means obtained for body weight (P) data were significantly different. The statistical analysis indicated that the averages for the morphological characteristics of tambaqui have significant effects, as expressed by p-value lower than 0.05 (Table 1). Analyses from the residues presented normal distribution with significantly small deviations.

Mathematical modeling was applied to evaluate the body growth pattern of tambaqui through the measurements collected for P, TC, CT and AC (Table 1) along 240 days of cultivation and to build projection curves. The Gompertz model considers that at 0 days, the animal already has weight and dimensions, thus it could consider the fish fry as a starting point for body growth evaluation of *C. macropomum*. The model demonstrated good fit to the experimental data at curve (Figure 2), indicating that it adequately represents tambaqui's growth pattern based on several variables.

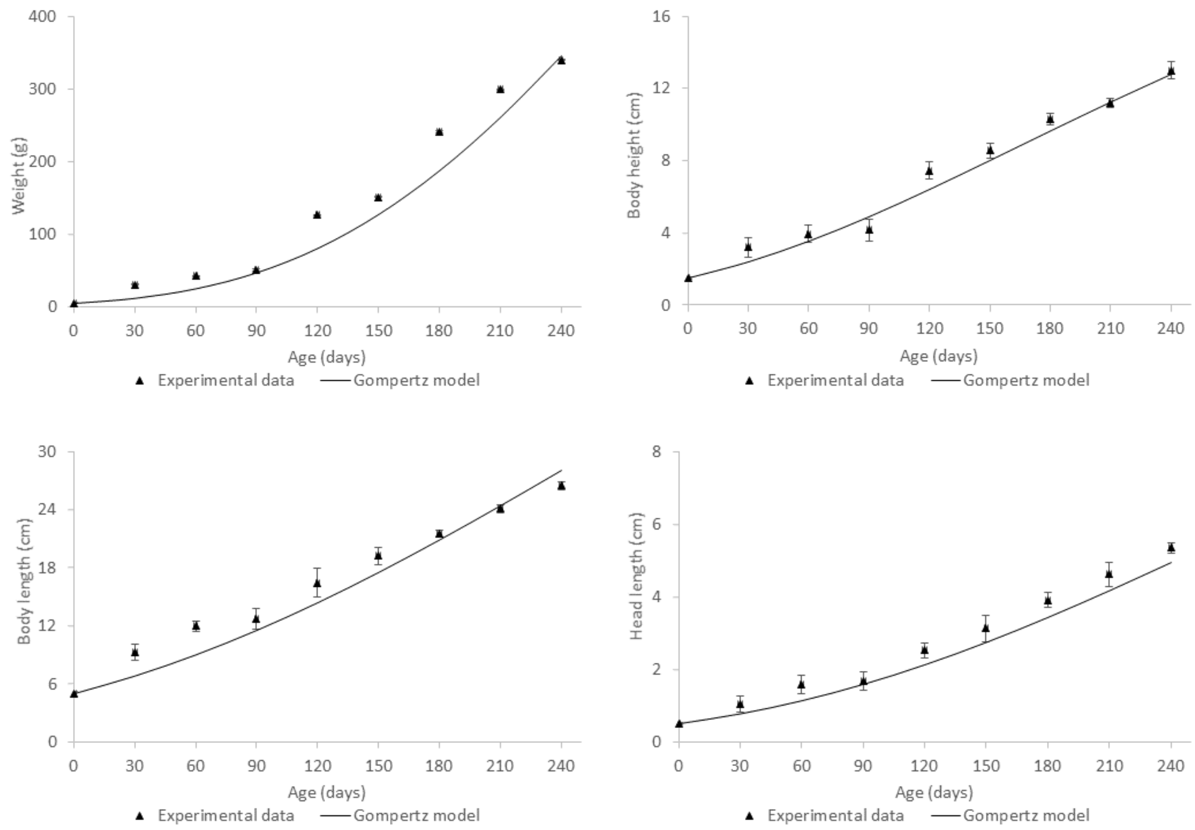

Figure 2. Adjustment of Gompertz model to the experimental data obtained for (A) body weight (P); (B) body length (CT); (C) body height (AC); (D) head length (TC) of tambaqui over time.

The data that better adjusted to Gompertz model curve was CT, followed by AC, P and TC. The distance of some data points from the curve could be explained by variations in environmental conditions (*e.g.*, rainy or drought seasons), feeding process, age (days of cultivation), gender or genetic characteristics, which can influence growth and fattening of the fish during the sampling period (Ibáñez-Escriche & Blasco, 2011; Mesquita, 2013; Silva & Fujimoto, 2015).

During the growing phase, the proportions and weight of the fish raised markedly until the inflexion point, when the growth rate started to stabilize. The fish showed body development until the 8th month, which hindered the experimental determination of the inflexion point for P, TC, CT and AC. Despite this, it was observed that the growth was equivalent for all parameters, when compared to the model’s curve. This suggests correlation among the morphometric characteristics of tambaqui, which can be used to define its growth patterns (Costa, 2011). The estimative of its standard growth, disregarding gender, give precedents to analyze the farming techniques applied in a cultivation system.

The determination of tambaqui growth pattern, in body dimensions, would provide more knowledge about its morphology, which could be useful to determine an optimal slaughter point. Also, it would be possible to define if it is the fish filet or the carcass, for example, that better represents the slowdown in the growth rate of *C. macropomum*. Moreover, the use of mathematical models could provide an estimation of tambaqui’s development until the moment of slaughter by outlining a growth curve that is representative for the entire population and predicts the inflexion point from a variety of characteristics (Figure 3).

Currently, the minimum weight considered good for tambaqui slaughter is 1.4 kg. However, the market preference is for fish that weight more than 5 kg (De Mello et al., 2015) Mathematical modeling is an important tool to predict when tambaqui will reach the desired slaughter weight based on age and characteristics that indicate the growth pattern of individuals under same farming conditions. Also, projection curves decrease the need of fish measurements for long periods of time and allow simulations of tambaqui corporal development from different regions or even of fish from different species, as has been applied for tilapia (De Lima Amancio et al., 2014; Santos, Freitas, Silva & Freato, 2007).

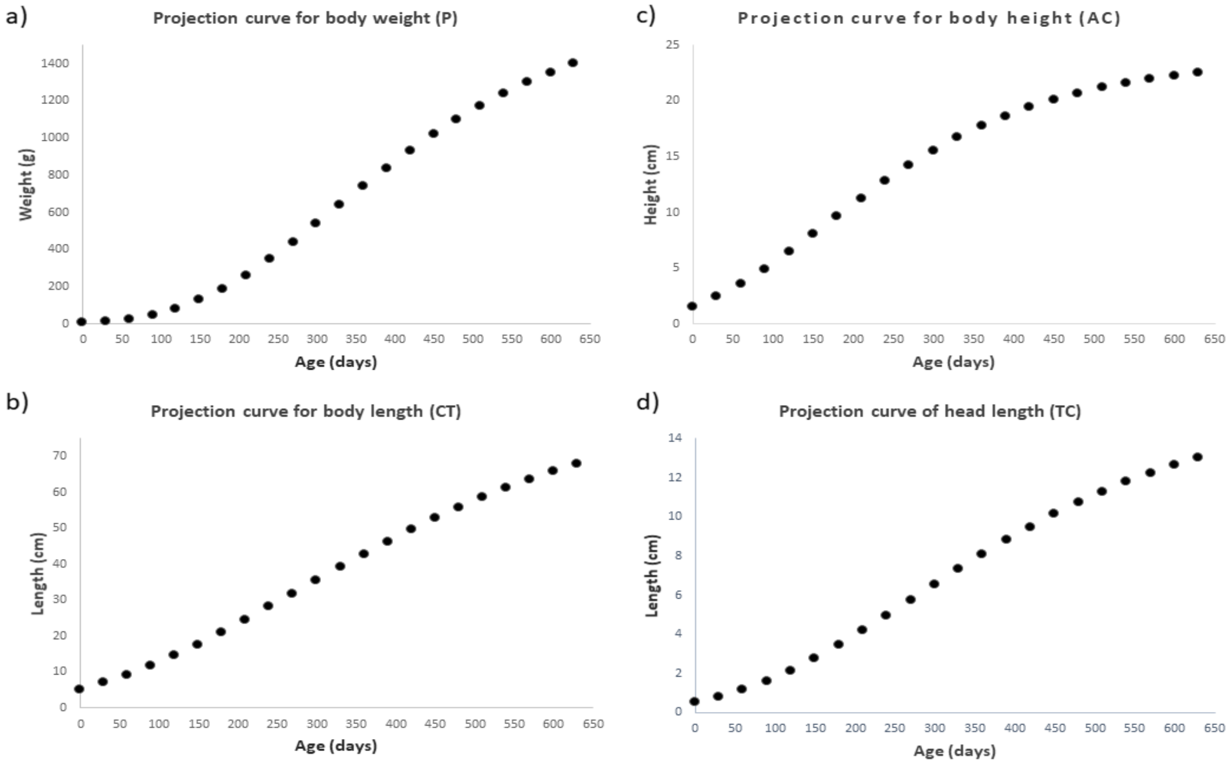

Figure 3. Mathematical projections built through the application of Gompertz method for different growth parameters of tambaqui.

The projections (Figure 3) were built using as maximum values for P, TC, CT, AC and age the described in literature as minimum requirements of tambaqui slaughter. The experimental data for P, TC, CT, AC and age (up to 230 days) was also considered in the construction of the curves. Inflexion points were identified through reductions in the value of *r* for each projection. The prediction of the inflexion point indicates that tambaqui's growth stops increasing at different ages.

Considering the mathematical model adjustment (Figure 2) and the projection curves (Figure 3), tambaqui cultivated in net-tanks in the state of Tocantins presented an increase in P earlier than in AC, TC and CT, suggesting that the latter are characteristic of its late development. This indicates that tambaqui's corporal growth pattern follows its weight gain, which could go up to 30 kg (De Mello et al., 2015). The curves also point out that the fish would keep growing after reaching 1.4 kg or after 630 days of cultivation, the common indicatives for slaughter.

Furthermore, tambaqui's growth curves indicate that they would keep developing and all the considered features would increase if slaughter was not executed. Thus, the fish could reach 30 Kg weight and 90 cm of body length as described in the literature, but there is no description of maximum values of head length and body height (Da Silva Oliveira et al., 2013; De Mello et al., 2015). The projections highlight the relevance of mathematical models in predicting growth patterns when the optimization of production is aimed. It could facilitate the selection of cultivation or genetic enhancement techniques that could distance the inflexion point, allowing a longer corporal development. Thus, such parameters should be studied in order to find a better approach to obtain fish with the desired features in a shorter period of time.

De Mello et al. (2015) study presents tambaqui's growth rate for larger fish, while ours considers the fish from fry phase until slaughter. The difference in the sampled animals' developmental stage could justify the discrepancy observed between the growth rates for P, TC, AC and CT (Table 2).

Table 2. Comparison of tambaqui's growth rate of morphological characteristics with data from the literature.

Features	Growth rate[1]	Literature[2]
Weight (P)	0.0054	0.0050
Head length (TC)	0.0048	0.0054
Body height (AC)	0.0062	0.0094
Body length (CT)	0.0038	0.0059

[1]Growth rate of the parameters used in the model based on the experimental data; [2]growth rate of the parameters from De Mello et al., 2015.

Comparing CT parameter, which is estimate to reach maximum length when the tambaqui is 20 years old, from different studies, it was observed that there is a low increase in CT since the age of 15 (Table 3).

Table 3. Estimative for body length (CT) using different data sets.

References	CT (cm)
Petrere, 1983	107.3
Isaac & Ruffino, 1996	118.5
Costa, Barthem & Villacorta-Corrêa, 1999	107.4
Penna, Villacorta-Corrêa, Walter & Petrere-Jr, 2005	85.1
Costa et al., 2013	90.4
This study	87.6

Conclusions

The growth pattern of tambaqui cultivated in net-tanks was well represented by the Gompertz model for all parameters (P, CT, AC and TC). The growth pattern curves suggest that the body weight of tambaqui is the feature that precedes the development of the other morphological characteristics. Projection curves indicate that the fish development would increase after reaching the standard features for slaughter because no inflexion point could be predicted within 630 days of farming for neither body weight (P), body length (CT), body height (AC) and head length (TC). By mathematically describing the dynamics of *C. macropomum* corporal growth, this study could be applied for different fish-farms using a captive culture system and, therefore, evaluating or developing novel strategies to optimize the production of tambaqui.

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